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OPTIMUM ORDER QUANTITY FOR DETERIORATING ITEMS IN LARGEST LIFETIME WITH PERMISSIBLE DELAY PERIOD

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ABSTRACT

In this research paper we develop an optimum order quantity model including power pattern demand, deterioration, and lifetime and delay period. In the proposed model deterioration is inversely proportional to time. There is maximum life time of product and supplier permit to retailer a certain time limit for delay the revenue. After this time interest is charged. There is particular cycle time. A numerical example is also presented in this paper.

KEYWORDS: Deterioration, Holding Cost, Ordering Cost, Purchase Cost, Maximum Life Time, Permissible Delay in Payment, Cycle Time, Interest Earned and Interest Charged

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INTRODUCTION

In the past research many inventory model has been developed with and without deterioration. In the practical situation many products deteriorate time to time, which is manufactured like food items, chemicals, Pharmaceuticals, photography, papers etc. and many of products has expiry date. The demand rate may be constant, time dependent, stock level dependent or price dependent etc. We have limited space for maintain the stock level. Due to low space, the order has to place frequently that increases the cost. Now if we store the large number of items for future demand then damages of items are possible due to many factors like storage conditions, weather condition, insects biting ete. Sometime supplier present attractive policies to large sale (like gifts or permissible time limit for payment). If supplier permits for delay in payments then after this time he will charge high interest from retailer.

The first attempt to describe optimal polices for deteriorating items was made by Ghare and Schrader [1], by EOQ model assuming exponential decay. Covert and Philip [2] developed an inventory model for two parameter weibull distribution for constant demand rate without shortages. Goswami and Chaudhuri [3] have developed an EOQ model for deteriorating items with shortages and a linear trend in demand. Hariga [4] gives an optimal EOQ models for deteriorating items with time varying demand. Teng, Chang, Dye and Hung [5] develop an optimal replenishment policy for deteriorating items with time varying demand and partial backlogging. Goyal [6] has given economic order quantity model under condition on permissible delay in payments. Aggarwal and Jaggi [7] have developed ordering policies of deteriorating items with permissible delay in payments. Jamal, Sarkaer and Wang [8] have introduced an ordering policy for deteriorating items with allowable shortage and permissible delay in payments. Soni and Shah [9] have developed an optimal order policy for stock dependent demand under progressive payment scheme. Teng, Krommyda, Skouri and Lou [10] have given a comprehensive extension of optimal ordering policy for stock dependent demand under progressive payment scheme. Teng, Min

and Pan [11] has developed economic order quantity model with trade credit financing for non decreasing demand. Sarkar [12] has developed an EOQ model for payment delay and time varying deterioration rate. Singh and Shrivastava [13] introduced an EOQ model for perishable items with stock dependent selling rate and permissible delay in payment and partial backlogging. Mishra S. S. and Mishra P. P. [14] investigate price determination for an EOQ model for deteriorating items under perfect competition. Sharma and Preeti [15] developed optimum ordering interval for random deterioration with selling price and stock dependent demand rate and shortages.

In the present model we develop an optimum order quantity model with power pattern demand and deterioration. There is maximum life time of product and supplier permit to retailer a certain time limit for delay the revenue. After this time interest is charged. And numerical example is also available.

Assumptions and Notations

- $C_0 =$ Ordering cost per order.
- $C =$ Purchasing cost per unit.
- $S =$ Sailing price.
- $C_h =$ Holding cost per unit.
- $I_c =$ Interest charged per year in stocks by suppliers.
- $I_e =$ Interest earned by investment per year.
- $I(t) =$ Inventory level at time't'.
- $\theta(t) =$ Deterioration rate dependent on time $= \dfrac{1}{1+m-t}, 0 \le t \le T \le m$.
- $m =$ Maximum life time in years of item.
- $T_d =$ Supplier permissible delay period.
- $D = -d\left(\dfrac{t}{T}\right)^{1/n}$ Power pattern demand.
- $T =$ Cycle time
- $TC =$ Total cost per year.

Mathematical Model

$$\frac{d}{dt}I(t) + \frac{1}{1+m-t}I(t) = -d\left(\frac{t}{T}\right)^{1/n}, \quad 0 \le t \le T \tag{1}$$

Boundary condition is $I(T)=0$ (2)

Now solution of differential equation (1) is

$$I(t) \quad = \quad -d\,\frac{(1+m)(1+m-t)}{T^{1/n}}\left[\frac{t^{1/n+1}}{\frac{1}{n}+1}+\frac{t^{1/n+2}}{(m+1)\left(\frac{1}{n}+2\right)}\right]+A$$

Using boundary condition (2), we have

$$I(t) \quad = \quad d\,\frac{(1+m)(1+m-t)}{T^{1/n}}\left[\frac{n}{n+1}\left(T^{1/n+1}-t^{1/n+1}\right)+\frac{n}{(1+2n)(m+1)}\left(T^{1/n+2}-t^{1/n+2}\right)\right] \tag{3}$$

Now holding cost

$$HC \quad = \quad C_h\int_0^T I(t)dt = \quad C_h d(1+m)nT^2\left[\frac{m+1}{1+2n}+\frac{1}{2(1+3n)}T^3-\frac{1}{2(m+1)(1+4n)}T^2\right]$$

Case 1: if $T \le T_d$

In this case retailer earned interest per cycle with return rate I_e. Therefore the annual interest earned per cycle is given as

Interest earned = $SI_e\left[\int_0^T Dtdt + DT(T_d-T)\right] \quad = \frac{SI_e}{1+2n}\left[T-T_d+n(T-2T_d)\right]dT$

Now the total cost per unit time

$TC_1=\frac{1}{T}$ (Purchasing cost + ordering cost + holding cost) – Interest earned

$$TC_1=\frac{1}{T}\left[\frac{d}{T^{1/n}}(1+m)^2\left\{\frac{n}{n+1}T^{(1+1/n)}+\frac{n}{(1+2n)(m+1)}T^{(2+1/n)}\right\}\right.$$
$$\left.+C_0+C_h d(1+m)nT^2\left\{\frac{m+1}{1+2n}+\frac{1}{2(1+3n)}T-\frac{1}{2(m+1)(1+4n)}T^2\right\}\right]$$
$$-\frac{SI_e}{1+2n}\left[T-T_d+n(T-2T_d)\right]dT$$

Now to find optimal cycle time T^*, $\frac{d}{dT}(TC_1)=0$ and $\frac{d^2}{dT^2}(TC_1)>0$

$$\frac{d}{dT}(TC_1)=\frac{d(1+m)n}{1+2n}-C_0T^{-2}+C_h d(1+m)n\left\{\frac{m+1}{1+2n}+\frac{1}{1+3n}T-\frac{3}{2(m+1)(1+4n)}T^2\right\}$$
$$-\frac{SI_e d}{1+2n}\left[2T-T_d+n(2T-2T_d)\right]=0$$

$$\frac{d^2}{dT^2}(TC_1)=2C_0T^{-3}+C_hd(1+m)n\left\{\frac{1}{1+3n}-\frac{3}{(m+1)(1+4n)}T\right\}-\frac{2SI_ed}{1+2n}[1+n]$$

Case 2: if $T_d < T$

Now interest charged per cycle is given as

Interest charged $=CI_c\int_{T_d}^{T}I(t)dt$

$$=CI_cd(1+m)n\left[\frac{m+1}{1+2n}T^2+\frac{1}{2(1+3n)}T^3-\frac{1}{2(m+1)(1+4n)}T^4-TT_d\left\{\frac{m+1}{n+1}\right.\right.$$

$$\left.+\frac{1}{1+2n}T-\frac{1}{2(n+1)}T_d-\frac{1}{2(1+2n)(m+1)}TT_d\right\}$$

$$\left.+\frac{n}{T^{(1/n)}}T_d^{(2+1/n)}\left\{\frac{m+1}{(n+1)(1+2n)}-\frac{nT_d}{(n+1)(1+2n)(1+3n)}-\frac{T_d^2}{(1+2n)(1+4n)(m+1)}\right\}\right]$$

And interest earned in time 0 to T_d $=sI_e\int_0^{T_d}Dtdt \quad =\frac{-sI_ednT_d^{(2+1/n)}}{(1+2n)T^{1/n}}$

Now the total cost per cycle is

$$TC_2=\frac{1}{T}\left[C_0+\frac{d}{T^{1/n}}(1+m)^2\left\{\frac{n}{n+1}T^{(1+1/n)}+\frac{n}{(1+2n)(m+1)}T^{(2+1/n)}\right\}+C_hd(1+m)nT^2\left\{\frac{m+1}{1+2n}\right.\right.$$

$$\left.\left.+\frac{1}{2(1+3n)}T-\frac{1}{2(m+1)(1+4n)}T^2\right\}\right]$$

$$+\mathrm{CI_c dn(1+m)T^2}\left\{\frac{\mathrm{m+1}}{\mathrm{1+2n}}+\frac{1}{2(1+3\mathrm{n})}\mathrm{T}-\frac{1}{2(\mathrm{m}+1)(1+4\mathrm{n})}\mathrm{T}^2\right\}$$

$$-TT_d\left\{\frac{m+1}{n+1}+\frac{1}{1+2n}T-\frac{1}{2(1+n)}T_d-\frac{1}{2(1+2n)(m+1)}TT_d\right\}$$

$$+\frac{n}{T^{(1/n)}}T_d^{(2+1/n)}\left\{\frac{m+1}{(n+1)(1+2n)}-\frac{nT_d}{(n+1)(1+2n)(1+3n)}-\frac{T_d^2}{(1+2n)(1+4n)(m+1)}\right\}$$

$$\left.+\frac{-sI_ednT_d^{(2+1/n)}}{(1+2n)T^{1/n}}\right]$$

Now to find optimal cycle time T^*, $\frac{d}{dT}(TC_2)=0$ and $\frac{d^2}{dT^2}(TC_2)>0$

$$\frac{d}{dT}TC_2 = -\frac{C_0}{T^2}+\frac{nd(1+m)}{1+2n}-C_h d(1+m)n\left\{\frac{m+1}{1+2n}+\frac{1}{1+3n}T-\frac{3}{2(m+1)(1+4n)}T^2\right\}$$

$$+CI_c d(1+m)n\left[\frac{2(m+1)}{1+2n}T+\frac{3}{2(1+3n)}T^2-\frac{2}{(m+1)(1+4n)}T^3-\frac{m+1}{n+1}T_d\right.$$

$$-\frac{2}{1+2n}T_dT+\frac{1}{2(n+1)}T_d^{\,2}-\frac{1}{(1+2n)(m+1)}TT_d^{\,2}$$

$$-\frac{1}{T^{(1+1/n)}}T_d^{(2+1/n)}\left\{\frac{m+1}{(n+1)(1+2n)}\right.$$

$$\left.\left.-\frac{nT_d}{(n+1)(1+2n)(1+3n)}-\frac{T_d^{\,2}}{(1+2n)(1+4n)(m+1)}\right\}\right]-\frac{sI_e dT_d^{(2+1/n)}}{(1+2n)T^{(1+1/n)}}=0$$

$$\frac{d^2}{dT^2}TC_2 = \frac{2C_0}{T^3}+C_h d(1+m)n\left\{\frac{1}{1+3n}T-\frac{3}{(m+1)(1+4n)}T\right\}$$

$$+CI_c d(1+m)n\left[\frac{2(m+1)}{1+2n}+\frac{3}{(1+3n)}T-\frac{6}{(m+1)(1+4n)}T^2-\frac{2}{1+2n}T_d-\frac{1}{(1+2n)(m+1)}T_d^{\,2}\right.$$

$$\left.+\frac{(1+1/n)}{T^{(2+1/n)}}T_d^{(2+1/n)}\left\{\frac{m+1}{(n+1)(1+2n)}-\frac{nT_d}{(n+1)(1+2n)(1+3n)}-\frac{T_d^{\,2}}{(1+2n)(1+4n)(m+1)}\right\}\right]$$

$$-\frac{(1+1/n)sI_e dT_d^{(2+1/n)}}{(1+2n)T^{(2+1/n)}}$$

Numerical Example

There is numerical example to test the model and its solution. We take some values as constant and study the effect on total cost total cycle time T and one of parameter.

Table & Graph 1: In case 1, Parameters are C = 10 Rs, S = 20 Rs, n = 1, C_h = 5 Rs, I_e = 0.5 Rs, I_c = 1 Rs, m = 0 and T_d = 4

d	T	TC1
1	2.68995	78.61712
1.2	2.71696	90.38539
1.4	2.73555	102.203
1.6	2.74912	114.0496
1.8	2.75948	125.9144

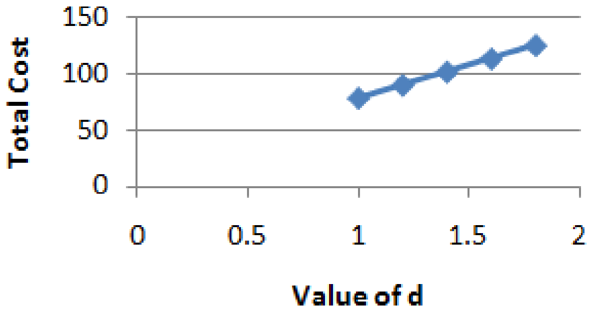


Table & Graph 2: In case 1, Parameters are C = 10 Rs, S = 20 Rs, d = 1, C_h = 5 Rs, I_e = 0.5 Rs, I_c = 1 Rs, m = 0 and T_d = 4

n	T	TC1
1	2.68995	78.61712
1.2	2.80057	79.35602
1.4	2.89207	79.924
1.6	2.96946	80.36125
1.8	3.03603	80.69828

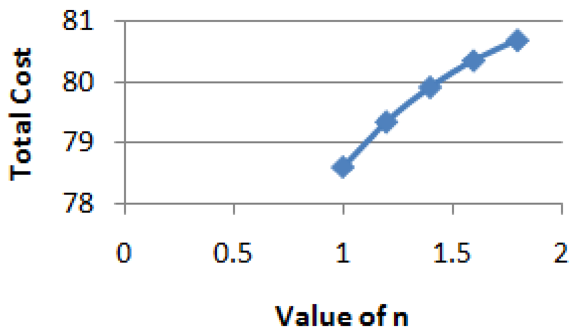


Table & Graph 3: In case 1, Parameters are C = 10 Rs, S = 20 Rs, d = 1, C_h = 5 Rs, I_e = 0.5 Rs, I_c = 1 Rs, n = 1 and T_d = 5

m	T	TC1
1	3.36945	102.8314
2	3.57908	128.8605
3	3.86802	172.0891
4	4.23682	237.4521
5	4.68528	331.2931

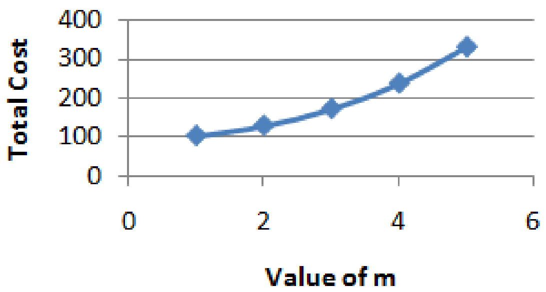


Table & Graph 4: In case 2, Parameters are C = 10 Rs, S = 30 Rs, n = 1, C_h = 5 Rs, I_e = 2 Rs, I_c = 0.2 Rs, m = 0 and T_d = 5

d	T	TC_2
1	7.5707	141.8236
1.2	7.57213	168.624
1.4	7.57315	195.4251
1.6	7.57391	222.2275
1.8	7.57451	249.0277

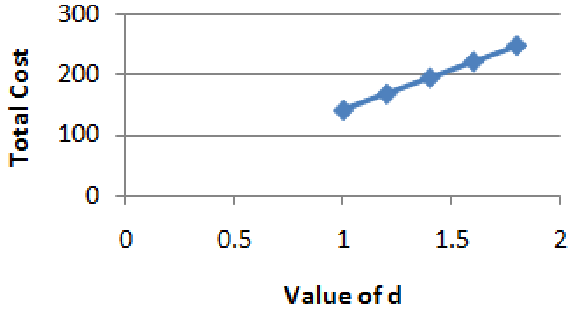


Table & Graph 5: In case 2, Parameters are C = 10 Rs, S = 30 Rs, d = 1, C_h = 5 Rs, I_e = 2 Rs, I_c = 0.2 Rs, m = 1 and T_d = 5

n	T	TC_2
1	6.93721	298.6796
1.01	6.79204	317.7401
1.02	6.62864	338.5892
1.03	6.43107	362.9443
1.04	6.12798	398.5383

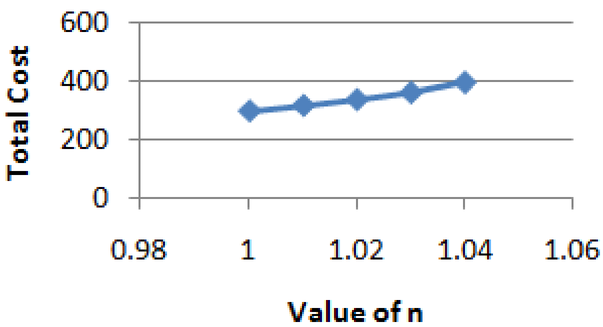
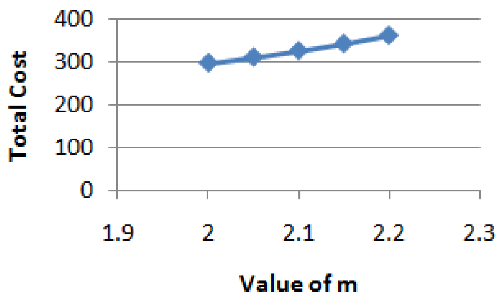


Table & Graph 6: In case 2, Parameters are C = 10 Rs, S = 30 Rs, d = 1, C_h = 5 Rs, I_e = 2 Rs, I_c = 0.2 Rs, n = 1 and T_d = 5

m	T	TC_2
2	6.93721	298.6796
2.05	6.85628	312.1355
2.1	6.76288	326.8439
2.15	6.65198	343.3024
2.2	6.51321	362.5585



CONCLUSIONS

In this model, we developed an optimum order quantity inventory model for deteriorating items with power pattern demand. In the model we study the effect of total cost and suitable cycle time with different parameters. In both

cases the total cost increases as the parameter d, n, m increases. Shortages are not allowed and no replenishment. In present model we use deterioration, demand, stock level, holding cost and deterioration cost. In practical situation there are many other factors which are responsible to change the total cost. This model can extended with other effective factors and it could be done in future research.

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